This is a story about curiosity and a story of observation, of why things happen, of peculiar outcomes, and of using the tools of science to find a solution to a problem.

Wind and water, compressible and non-compressible fluids, provide some of the deepest mysteries in mathematics, but researchers past and present give us formulae to work with, at our peril, it must be said.

Pressures and forces, velocities and vectors are complex and involve squares and square roots and quadratics and surds. The classic solution to quadratic equations uses terms like '√' and '±', bringing fear to teenagers and octogenarians the world over. A week ago, paradoxically, that quadratic solution to v_{sW} and v_{sT} magically simplified to a pair of equations containing plain old densities. The connecting link between bearings β_{nom} and β (see their definitions in the following paragraphs) involving months of conjecture, study of texts, and pages of spread-sheets just fell into place.

Tide-flows are stronger at the southern end of Port Phillip and sometimes the apparent wind (wind vector minus tide vector) drops sufficiently to deny a fleet's ability to manoeuver. Apparent wind is such a critical factor in setting the course axis, the start-line, and even the type of course, that it is vital that a Race Officer is aware of the forecast and actual winds and tide-flows during the course of the race.

The wind range that we accept for racing is 5kt to 25kt, generally from the south, with occasional summer breezes from anywhere. Tide-flow is quite variable in direction, east-west in the southwest half of our racing area, and northwest-southeast in the remainder. Tidal strength can vary too, but it will generally will not exceed 2kt or 1m/s -a brisk walk. Marching pace is ⅞ m/s.

One of the interesting aspects of sailing here is that competitors can experience different apparent winds with no certainty as to whether the cause be wind or tide-flow or the 'nut on the tiller'.

Does a support vessel give unsatisfactory measures of apparent wind due to its above-water or its below-water characteristics?

When there's no tide-flow, an immersed object will move downwind slower the more it's immersed. There are two limits to this case - if it's barely immersed it still won't move as fast as the wind, and if it's fully immersed the wind has no effect at all. (Skipping over the water is outside the range because then none of it is immersed.) Conversely, in a tide-flow and dead-calm air, a fully immersed object will move down-tide as fast as the tide. But the less it's immersed, air resistance on the un-immersed part will slow it down, so that the object won't move quite as fast as the tide-flow.

A semi-submerged sphere, being axi-symmetric, has equal projected areas A_{air} and A_{water} at right angles to the wind and tide-flow forces and reactions acting above and below the water-line. Its drag characteristics, aerodynamic or hydrodynamic shape factors C_{fig} are equal. Moreover, its response to any wind or tide-flow direction is the same.

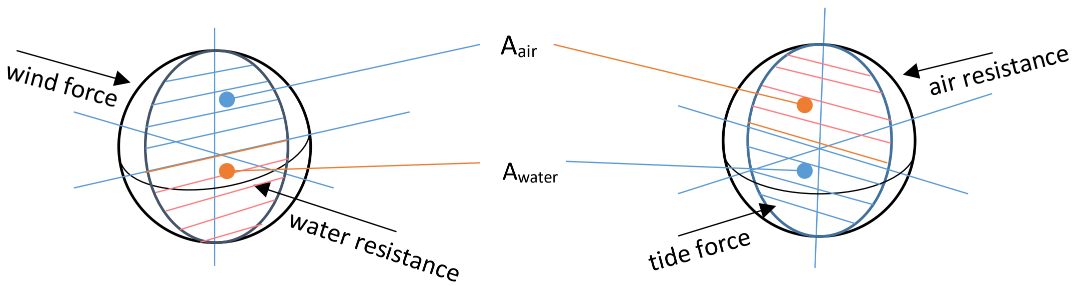

Fig. 1. Wind and tide-flow on a semi-submerged sphere.

Consider a wind V_W and tide-flow of zero on a sphere. The sphere's speed v_{sW} due to wind is

$v_{sW} = V_W \sqrt{(\rho_{air})} / [\sqrt{(\rho_{water})} + \sqrt{(\rho_{air})}]$ where the density of air ρ_{air} is 1.2kg/m^3 and the density of water ρ_{water} is 1000kg/m^3

$= V_W \sqrt{1.2} / [\sqrt{1000} + \sqrt{1.2}] = 0.0335\, V_W$

For a 20kt wind, the half-submerged sphere moves through the water at 0.67kt.

Consider zero wind and a tide-flow V_T kt. The sphere's speed due to the tide-flow is

$v_{sT} = V_T \sqrt{(\rho_{water})} / [\sqrt{(\rho_{water})} + \sqrt{(\rho_{air})}]$ and, using the appropriate densities,

$v_{sT} = V_T \sqrt{1000} / [\sqrt{1000} + \sqrt{1.2}] = 0.9665\, V_T$ giving 1.933kt for a 2kt tide-flow.

When there is both wind and tide-flow, the speed vectors can be combined and there are some interesting outcomes. The apparent wind strength v_{app} and its bearing β that would be experienced by the sphere is the vector difference between the true wind V_W and the sum of the vectors v_{sW} and v_{sT} that have just been determined. The nominal apparent wind strength $v_{app\ nom}$ and its bearing β_{nom} is the vector difference between the true wind V_W and the tide-flow V_T.

Consider a 20kt southerly wind and a 2kt tide-flow changing from north to south through west in 45° steps. The table below shows the outcomes for each case.

V_W 20kt south wind					
Tide-flow 2kt	to 360°	to 315°	to 270°	to 225°	to 180°
$v_{app\ nom}$ β_{nom}	18.0 from 180°	18.6 from 184.4°	20.1 from 185.7°	21.5 from 183.8°	22kt from 180°
v_{app} β	17.4 from 180°	18.0 from 184.4°	19.4 from 185.7°	20.7 from 183.8°	21.3 from 180°

For each tide-flow direction, the sphere experiences a lesser apparent wind speed <u>but its bearing exactly matches β_{nom}</u>. This is the case for wind-speeds of 15, 10 and 5kt, and it's quite unexpected.

An axisymmetric object <u>whose submergence varies</u> requires the definition of the above and below waterline parameters, rather than nominating a value of 50%.

Let $\alpha = A_{water} / A_{air}$ and $\lambda = C_{fig\ water} / C_{fig\ air}$ and then the expression for v_{sW} becomes

$$v_{sW} = V_W \sqrt{(\rho_{air})} / [\sqrt{(\alpha\, \lambda\, \rho_{water})} + \sqrt{(\rho_{air})}]$$

and the expression for v_{sT} becomes

$$v_{sT} = V_T \sqrt{(\alpha\, \lambda\, \rho_{water})} / [\sqrt{(\alpha\, \lambda\, \rho_{water})} + \sqrt{(\rho_{air})}]$$

These expressions seem to be more complex, but their algebraic form enables the opportunity to prove that there is parity between β and β_{nom}. That involves a few lines of tedious algebraic manipulation, but it appears to be true for all axisymmetric floating objects. Essentially there is a coupling between above and below waterline parameters that balance wind and tide effects.

The case of non-axisymmetric objects requires some 'thinking outside the box'.

The equilibrium equations for v_{sW} and v_{sT} need to be related to x and y axes, with a vessel that has properties α_x, α_y, λ_x, λ_y, and $A_{air\ x}$, $A_{air\ y}$, $A_{water\ x}$, $A_{water\ y}$, $C_{fig\ air\ x}$, $C_{fig\ air\ y}$, $C_{fig\ water\ x}$ and $C_{fig\ water\ y}$.

When the vessel is subjected to a wind V_W and a tide-flow V_T, these can each be resolved into the vessel's x and y directions as V_{Wx} and V_{Wy} and V_{Tx} and V_{Ty}. The response of the vessel to the wind are velocity vectors $v_{sw\,x}$ and $v_{sw\,y}$ and to the tide-flow are $v_{sT\,x}$ and $v_{sT\,y}$. As before, the difference between them and the wind vector V_W is the apparent wind vector v_{app}.

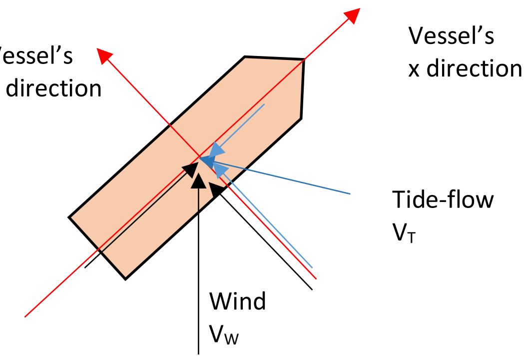

Fig. 2. Non-axisymmetrical vessel subject to wind and tide-flow – the vessel's heading is the x-direction.

This has been done for a number of theoretical vessels that might correspond with our support vessels. The results are different from the axisymmetric cases, but they do not differ by a significant amount. Further, if the vessel's x and y properties equal those of an axisymmetric body, the results are the same, confirming the validity of the process.

Rather than present a table of outcomes, a visual plot is shown of values of apparent wind v_{app} in a wind and a tide-flow as the vessel alters its heading. An example is presented of a vessel's recorded theoretical apparent wind-strengths and bearings.

The vessel is adrift at headings that are multiples of 30°. The true wind is a 20kt southerly and the tide-flow is 2kt to 315°. The vessel's dimensions are 4m length, 2m beam, 1m freeboard and 0.5m draft. Its shape characteristics are estimated as $C_{fig\,x\,air}$ 1.4 ; $C_{fig\,x\,water}$ 0.3 ; $C_{fig\,y\,air}$ 1.2 ; $C_{fig\,y\,water}$ 1.8. The nominal apparent wind is 18.64kt from 184.35°.

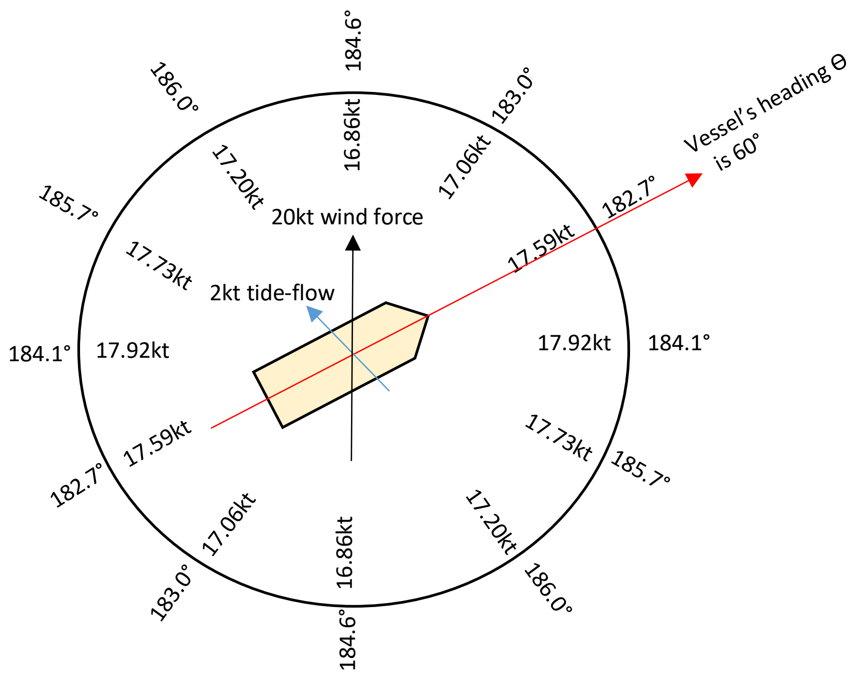

Fig. 3. Vessel L_x = 4m Ly = 2m draft = 0.5m - v_{app} readings in 180° wind and 315° tide at 30° heading intervals.

Conversely, when a vessel is chosen with radically variant x and y properties, such as a wall-like superstructure parallel with the keel and an underwater shape broad of beam and short in waterline length, the theoretical readings vary markedly as the vessel changes its heading, or when the wind and the tide-flow values change.

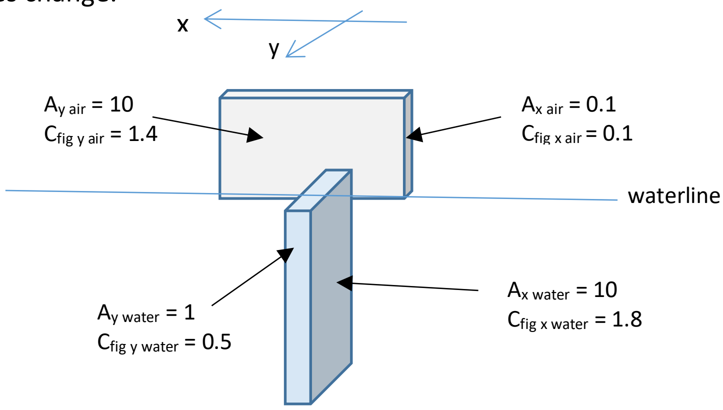

Fig. 4. A radically different assymetrical object

Conclusions.

- An axisymmetric floating object, such as a sphere or an upright cylinder, would record apparent wind strengths that do not exceed the nominal apparent wind $v_{app\ nom}$, depending upon how deeply it is immersed. A half-submerged object will be more affected by the wind than the tide and will record a smaller value of v_{app}. A fully submerged object will be affected only by the tide and will record a strength reading of $v_{app\ nom}$. Nevertheless, at any degree of immersion, the recorded bearing is β_{nom}.

- If, for some reason, a support vessel has axisymmetric characteristics, depending upon its above-water characteristics compared to those under-water, the recorded apparent wind strengths will be less than $v_{app\ nom}$, but the bearings will all be β_{nom}.

- A support vessel will usually have non-axisymmetric characteristics, although generally symmetric about its y-axis (the axis parallel to its keel). Estimates of typical support vessels' properties show that acceptable estimates of apparent wind will be provided no matter what the heading of the vessel, and no matter what type of vessel is used. A confirmation is that the standard deviation of the theoretical apparent wind data shown in Fig. 3. Shows that a 0.8kt and 2½° variation might be expected to cover 95% of recorded values.

- A Race Officer's anxiety level is connected far more to the value of the apparent wind bearing rather than the strength. A change of course during the race is always in mind and prepared-for, while an abandonment or a restart due to wind-strength or lack of it is an almost routine procedure.

- The estimates of the strength of the apparent wind recorded might vary significantly depending upon the support vessel's 'windage'. Nevertheless, real-world variation in wind-strength readings are not only likely, they are inevitable. Furthermore, any other anchored support vessel is able to provide wind-strength readings that are independent of downwind drift.
- Research is needed to provide better estimates of A_x and A_y, and $C_{fig\ x}$, $C_{fig\ y}$ of our vessels.
I acknowledge the work done by Standards Association of Australia Committee (CA1170 – 2, 2011) members, past and present, particularly in terms of using the parameter C_{fig}, the shape factor pertaining to a body in a fluid stream.